SOLUZIONE ESONERO

① $\lim_{x\to 0} \frac{\sqrt[3]{1-x^2} - \sqrt[4]{1+x^2}}{x^3+x^2} = \lim_{x\to 0} \frac{(1-x^2)^{1/3} - (1+x^2)^{1/4} - 1 + 1}{x^3+x^2}$

$(1-x^2)^{1/3} - 1 \cong -\frac{1}{3}x^2$

$(1+x^2)^{1/4} - 1 \cong \frac{1}{4}x^2$

$x^3 + x^2 \cong x^2$

$\lim_{x\to 0} \frac{-1/3\,x^2 - 1/4\,x^2}{x^2} = -\frac{7}{12}$

② $\lim_{x\to 0} (1+x^2)^{\frac{2}{1-\cos x}}$

$\lim_{x\to 0} e^{\frac{2}{1-\cos x}\log(1+x^2)} = e^4$

$\lim_{x\to 0} \frac{2}{1-\cos x}\log(1+x^2) = \frac{2x^2}{1/2\,x^2} = 4$

$\log(1+x^2) \cong x^2$

$1-\cos x \cong \frac{1}{2}x^2$

③ $\lim_{x\to -\infty} \frac{e^x + \operatorname{sen} x + x^2}{\log x^2} = \lim_{x\to -\infty} \frac{x^2}{\log x^2} = \lim_{x^2=t\to +\infty} \frac{t}{\log t} = +\infty$

($x \to 0$, $\operatorname{sen} x \to 0$, $x^2 \to 0$)

$e^x + \operatorname{sen} x + x^2 \cong x^2$ per $x \to -\infty$

④ $\lim_{x\to -\infty} \sqrt{2x^2+3} + x = \lim_{x\to -\infty} \sqrt{x^2\left(2+\frac{3}{x^2}\right)} + x = \lim_{x\to -\infty} \left(-x\sqrt{2+\frac{3}{x^2}} + x\right) =$

$= \lim_{x\to -\infty} x\left(-\sqrt{2+\frac{3}{x^2}} + 1\right) = +\infty$

⑤ $\lim_{x\to \pi} \frac{x^2 - 2\pi x}{x-\pi}$ $\begin{cases} \lim_{x\to \pi^-} \frac{x^2-2\pi x}{x-\pi} = +\infty \\ \lim_{x\to \pi^+} \frac{x^2-2\pi x}{x-\pi} = -\infty \end{cases}$

$x^2 - 2\pi x > 0$ — 0 — 2π

$x - \pi > 0$ — — — — π

③ $\sqrt{1+x} + 2^x = 3^x - \operatorname{arctg}(x+1)$

$f(x) = \sqrt{1+x} + 2^x - 3^x + \operatorname{arctg}(x+1)$

$x+1 \geq 0 \quad x \geq -1 \quad A = [-1, +\infty[$

$f(1) = \frac{1}{2} - \frac{1}{3} > 0$

$\lim_{x \to +\infty} f(x) = 3^x \left(\frac{\sqrt{x}}{3^x} \to 0 + \left(\frac{2}{3}\right)^x \to 0 - 1 + \frac{\operatorname{arctg}(x+1)}{3^x} \to \frac{\pi}{2} \cdot 0 \right) = -\infty$

$\exists \alpha \in A \text{ T.c. } f(\alpha) = 0$